

## **A GLIMPSE OF PHYSICS**

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### **ABSTRACT**

Three stories are told, interspersed by two interludes. The first story is about Einstein's mass-energy equivalent, the second is on De Broglie's particle-wave duality, and the third is the perennial controversy of determinism versus probabilism. The first interlude gives Pythagorean theorem and Socratic anamnestic questions, whilst the second is a brief account of Philosophy of Science focused on paradigm and values. Through the stories and their interludes attempt is made to give just a glimpse of Physics.

**key words:** *science, physics, faith, values, sustainability*

### **1. STORY 1 : PLAGIARISM, EINSTEIN, AND PLANCK**

It was Friday, June 1<sup>st</sup>, 2012, in the Probowinoto Lecture Hall. There was a talk given by Dr. Henry Sulistyono Budi, on "Plagiarism: its legal and ethical aspects". The speaker is a lawyer specializing in copywrite and TRIPP (Trade-Related Intellectual Property rights Protection). When the interesting talk ended and the moderator opened the discussion session, I raised a question. I was wearing this cap, with Einstein's equation,  $E = \gamma m_0 c^2$ , on it, so while pointing to the equation I asked, whether I committed the sin of plagiarism and modification, as there was neither mention of Einstein, nor was there reference to his article, "*Zur Elektrodynamik Bewegter Koerper*," published in an issue of *Annalen der Physik*, in 1905. Besides, the original equation,  $E = mc^2$ , was modified to  $E = \gamma m_0 c^2$ .

### 1.1. Einstein and Planck

Of course you know that the equation,  $E = mc^2$ , expresses the equivalence of mass ( $m$ ) and energy ( $E$ ), where  $c$  is the speed of light in free space. In the equation on my cap,  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ ,  $\beta = \frac{v}{c}$ , and  $m_0$  is the rest mass of the particle under consideration. Either  $E = mc^2$ , or  $E = \gamma m_0 c^2$ , is another way of saying that the square of a vector is a scalar, — an invariant. In this particular case, the vector that is inner or “dot” multiplied by itself is the momentum-energy four vector,  $p_\mu = (\vec{p}, j \frac{E}{c})$ , where  $p_\mu = m_0 U_\mu$ ,  $U_\mu$  being the velocity four-vector, i.e. the proper-time derivative of the space-time coordinates  $x_\mu = (\vec{x}, jct)$ , written as  $U_\mu = \frac{dx_\mu}{d\tau} = \gamma \frac{dx_\mu}{dt}$ . Thus the fourth component of the momentum-energy four-vector is  $P_4 = m_0 U_4 = \gamma m_0 v_4 = \gamma m_0 \frac{d(jct)}{dt} = j \gamma m_0 c = j \frac{\gamma m_0 c^2}{c}$ , giving  $E = \gamma m_0 c^2$  or  $E = mc^2$ , since the mass of the relativistically moving particle is  $m = \gamma m_0$ .

Since scalars are invariant under a Lorentz transformation from an inertial frame of reference to another inertial frame of reference,

$$p^2 - \frac{E^2}{c^2} = 0 - \frac{E_0^2}{c^2}$$

where the non zero term on the right side is  $p_4^2$  in the rest frame of reference. In this frame of reference the velocity of the particle is 0, hence the square of its momentum is also 0. Since in this frame of reference  $\vec{v} = 0$ , then  $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0}} = 1$ , hence

$E_0 = m_0 c^2$  (the rest energy). The above equation can be rewritten as

$$E^2 = (pc)^2 + (m_0 c^2)^2,$$

which is the Pythagorean form of Einstein’s mass-energy equivalence. For photons, which are massless, i.e.,  $m_0 = 0$ , we have

$$E^2 = (pc)^2, \text{ or } p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}.$$

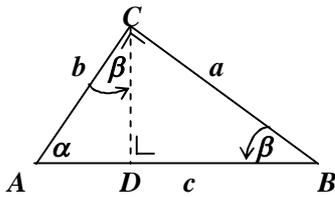
Use has been made of the Planck equation,  $E = hf$ .

The relation  $E = hf$ , for photons, was obtained by Max Planck in 1901, while  $E = mc^2 = \sqrt{p^2 c^2 + m_0^2 c^4}$  was first derived from his Special Theory of Relativity by

Albert Einstein, in 1905. It is valid for particles of any mass, i.e., for both massless as well as massive ones.

**1.2. Interlude 1: Pythagoras and Socratic Anamnesis**

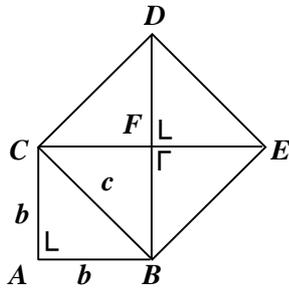
As I said,  $E^2 = (pc)^2 + (m_0c^2)^2$  is Einstein’s mass-energy equivalence in Pythagorean form. Pythagorean theorem states that in a right-angle triangle the square of the hypotenuse is equal to the sum of the squares of the right sides.



*AB*: base, *C*: top, *c*: hypotenuse,  
*a* & *b*: right sides,  $\alpha$  &  $\beta$ : base angles,  
angle  $\gamma = 90^\circ$

Figure 1

In his “**Dialogue (Meno)**”, Plato described how Socrates demonstrated to their friend, Meno, that a kid, the son of a slave, was able to prove Pythagorean theorem under the guidance of a series of anamnetic questions given by the master philosopher. The proof was only for an isosceles right-angle triangle.



The area of square *ABFC* = twice the area of isosceles right triangle *ABC* =  $b^2$ . The area of square *BCDE* = four times the area of triangle *ABC* or twice the area of square *ABFC*.

Figure 2

Thus  $c^2 = 2b^2 = b^2 + b^2$ , *Q.E.D.* (*quod erat demonstrandum*).

In the right-angle triangle *ABC* in which *C* is the top vertex and *AB* is the base (Fig. 1) the area =  $c^2f(\beta)$ , where *c* is the base and  $f(\beta)$  is a certain function of a base angle,  $\beta$ . It is so, because a right-angle triangle is well specified when its base and one of its base angles are given. Dimensionally the relation is correct as  $f(\beta)$  is dimensionless. Since triangles *ABC*, *ACD*, and *CBD* are all congruent to each other, then their areas are  $c^2f(\beta)$ ,

$b^2f(\beta)$ , and  $a^2f(\beta)$ , respectively. But it is obvious from the figure that  $\text{area } ABC = \text{area } ACD + \text{area } CBD$ , or

$$c^2f(\beta) = b^2f(\beta) + a^2f(\beta),$$

hence  $c^2 = b^2 + a^2$  (QED)

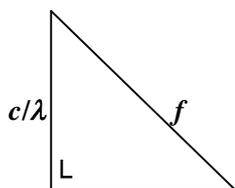
It can be easily shown that  $f(\beta) = \frac{1}{4}\sin(2\beta)$

## 2. STORY 2: FAMILY REUNION AND THE PRODIGIOUS PRINCE

On Sunday, June 17<sup>th</sup>, 2012 we had a family reunion in Yogyakarta. Before and after a solemn worship, we had a good time and a lot of fun, shaking hands, hugging, kissing, chatting, joking, bantering, and enjoying delicious cookies and having a good luncheon. When we — our grandson and granddaughter, their granny, and I — were leaving, two nephews of mine stopped me at the gate, saying “Hey, uncle Liek, what’s that on your cap? What does it mean?” They were referring to this equation,  $p = \hbar k$ . I answered: Forget it. You guys wouldn’t understand. This is definitely beyond your capacity!” But later I sent them an E-mail, explaining the equation by way of a story.

Once upon a time there was a young prince who belonged to a nobility family of considerable influence in France. His name was Louis, — Louis Victor de Broglie. He was a graduate of Sorbonne University, and was of the Humanities major with a specialization History. But he was captivated by Einstein’s Theory of Relativity, which he studied by himself. He argued, “If light has dual nature as a stream of particles and at the same time also as propagating electromagnetic wave, then why shouldn’t other particles? Then he made a bold step, generalizing the relation  $p = h/\lambda$  which had heretofore been considered valid for photons only.

He drew a right-angle triangle, putting frequency as its hypotenuse and the reciprocal of wavelength times the speed of light as one of its right sides. In analogy to the Pythagorean form of Einstein’s equation,  $E = mc^2$ , the other right side must be an invariant corresponding to Einstein’s rest energy,  $m_0c^2$ . This invariant of wave was as yet unnamed. You can show easily that this right-side of de Broglie’s triangle is  $m_0c^2/h$ .



unnamed  
invariant

Figure 3

He used his postulate of particle-wave duality as the corner stone of what was later developed into *La Mecanique Ondulatoire* (Wave Mechanics). Louis de Broglie then wrote a paper with the title “*Recherches sur la theorie des quanta*” (Researches on quantum theory) and submitting it to the Faculty of Sciences at the University of Paris, requesting that he be permitted to defend the paper as a doctoral thesis.

Professor Paul Langevin, a leading theorist at the university hesitated. On the one hand, Louis de Broglie was “a nobody” in Physics at the time. What if his accepting Louis de Broglie’s paper as a dissertation would make the University of Paris the laughing stock of the physicists in all over Europe? On the other hand, he did not dare to insult the highly-respected de Broglie family. So he showed Louis de Broglie’s paper to Einstein and asked for his opinion. Einstein said: “It may look crazy, but it really is sound.” Being thus encouraged, Langevin let Louis de Broglie defend his dissertation.

It surely is counter-intuitive to think that a particle, an electron, say, which is necessarily localized, could at the same time be a wave, which is always spread-out. Louis de Broglie couldn’t explain it. He characterized his theory as “a formal scheme whose physical content is not yet determined.” Anyway, he was deemed successful in his oral defense, and was awarded the degree of doctor in Physics. That was in the year 1924. The postulate of particle-wave duality was later confirmed experimentally by Clinton Davisson in USA and by George P. Thomson in UK. In 1929 Louis de Broglie was awarded Nobel prize. He was the first, and so far the only, person whose thesis made him a Nobel prize winner.

Two years before Louis de Broglie became a Nobel Laureate, at the Fifth Solvay Physics Conference, held in Brussels, October 23 – 29, 1927, his concept of the pilot wave associated with a particle was clobbered very hard, especially by Wolfgang Pauli. But de Broglie showed that by demanding that the wave associated with the electron in the Bohr’s theory of the hydrogen atom be of such wavelengths so as to make the circumference of the circular Bohr orbits an integral multiple of the wavelength, i.e.,  $C = n\lambda$ , one could do away with Bohr’s “*ad hoc*” quantization rule for the orbital angular momentum. Later, use was made of Planck’s equation,  $E = \hbar\omega$ , and that of de Broglie’s,

$p = \hbar k$ , by Erwin Schroedinger, who was then a professor at *ETH* Zuerich, to develop what we now know as Schroedinger equation,  $\left(-\frac{\hbar}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\delta\psi}{\delta t}$ .

The relation  $C = n\lambda$  for Bohr's orbits of the electron in hydrogen atom is the same as the condition satisfied by a standing wave between two nodal points. I applied this condition to an electron moving back and forth between the two "walls" of Coulomb potential well, and obtained the quantized energies of the Bohr hydrogen atom. In my actual derivation use was made of only one half of the potential well, i.e., from the wall to the symmetry axis of the well, so that the required condition to be satisfied was

$$\int_0^{r_m} \frac{ds}{\lambda} = \frac{n}{2}, r_m \text{ being the distance between the axis and the wall, from which}$$

$$E_n = -\frac{\mu Z^2 e^4}{32\pi^2 \epsilon_0^2 \hbar^2 n^2}, n = 1, 2, \dots$$

was obtained for a hydrogen-like atom of atomic number  $Z$  and reduced mass  $\mu$ .

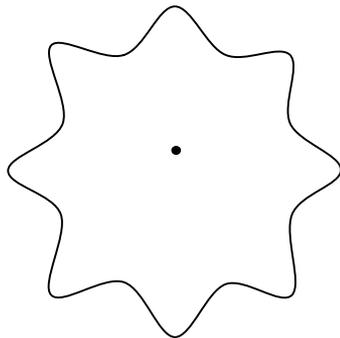


Figure 4 : De Broglie's picture of a Bohr orbit.

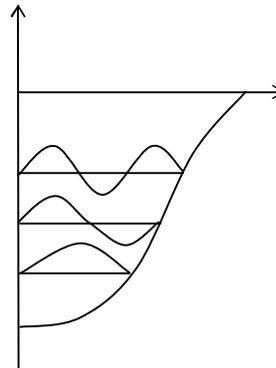


Figure 5

When saying to Langevin that de Broglie's idea was crazy but sound, Einstein was truly appreciative. In his letter to Hendrik A. Lorentz, dated December 16, 1924, which was learned of by de Broglie only after Einstein's death in 1955. Einstein wrote: "A younger brother of de Broglie (the one we know) has undertaken a very interesting investigation (Paris Dissertation, 1924) to interpret Bohr-Sommerfeld quantum rules. I believe this is a first weak ray to illuminate the most serious of our physical riddles."

## 2.1. Interlude 2: Science: Is It Epistemic?

According to Richard M. Bubes (1989?) science is a way of knowing. It is based on human interpretation of data which are publicly obtained through interaction with nature and the realities therein. Following Robert K. Merton, we use the term “science” here to refer to it not only as product, but also as process and paradigm. The process is the trail traversed by the scientists to arrive at the product, whereas the paradigm is the “guiding star used by the scientists so as not to get astray. It is an ordering belief framework. The codes of ethic that should be upheld in scientific endeavors are spelled out in four imperatives. These are universalism, communalism, disinterestedness, and organized skepticism.

But the third imperative, i.e., disinterestedness or detachment, is no longer a “decree” that *coute-que-coute* must be obeyed, both in the humanities and in “hard” sciences. In researches that use in-depth case study approach, the researcher deliberately blends her or himself with the case being studied, thus being accepted as and becoming part of the system. In the sub-atomic world, the very act of the observer observing the observed disturbs the latter and changes its state. Thus the classical picture of an observer standing aloof at a distance from the phenomenon has been drastically changed by the new conviction.

This shows that science is *not* objective, in the sense that it does not give us the absolute understanding of things, exactly as they really are. Science is also based on faith, — faith that realities in nature are orderly and to certain extent comprehensible, yet to the limited capacity of our mind they are so complex and even mysterious. According to the physicist Fritjof Capra and the theologian David Steindl-Rast, science is not objective; it is epistemic in the sense that it depends on, and limited by, our capacity to know.

I started this interlude by quoting Richard Bube that science is a way of knowing. In the scientific community there is no pretence that it is either the only way, or the best one. Using approximate metaphors and simplified models, science describes essential characteristic properties, events, and phenomena in the natural world, and provides insights into the answers to questions of how and why these events occur.

Great scientists like John Dalton and Albert Einstein advise us that the model, and thus also the theory that is developed from it, be based on as simple and as few basic assumptions as possible, in keeping with the principles of simplicity and parsimony. This “stinginess” in the assumptions must, however, be compensated with as much *Einfuehlung* as possible. *Einfuehlung* is a term used by Einstein. It is a sort of loving empathy towards the objects of your investigation. This is the recipe to open the possibility of reaching a universal principle as, Einstein believed, there is no logical path that leads to it.

Yes, universal principles render a part of the complex realities scientifically comprehensible, but there are also things that are very important and of the most relevance to us that cannot be explained by science. To peek into these mysterious things and get a glimpse of each of them, we turn to spiritualism, philosophy, or religion.

Metaphorically science can be thought of as an edifice sitting firmly on three foundational pedestals. These are ontology, epistemology, and axiology or teleology. The third pedestal sets the objectives<sup>1</sup> which science is in pursuit of, and they are reflected in those values<sup>2</sup> which the scientists uphold in the development and application of science. On the “development” side, the more relevant values are constitutive values, such as truth, honesty, openness, willingness to collaborate, and suspension of (dis)beliefs, whereas on the “application” side the more important values are contextual values which have a lot to do with the societal context in which scientific endeavors are decided upon and carried out. As Edward Teller (1999) said, “Ethics begins where science ... has opened possibilities (and offered choices)<sup>3</sup>.”

We must be really careful in deciding which ones of these alternatives that not only can, but also *may*, be developed further in applied research to design prototypes, and which ones of the chosen prototypes that may be perfected in technological R & D to end up in environmentally and user-friendly technologies. For instance, considering that plutonium is so fiendishly toxic and of “tough” radioactive longevity, I would argue whether the world can be sustainable if we adopt plutonium-based energy policy, unless

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<sup>1</sup> *telos* = objective

<sup>2</sup> *axios* = value

<sup>3</sup> The parentheses are mine

the skipper at the helm of our one-and-only ship, *GAIA*, is a fiercely authoritarian dictator.

Physics is a branch of science. It is one of the basic sciences, if not even **the** most basic of all sciences. Physics deals with events and phenomena in the inanimate world. As such, it shares with the rest of sciences the general characteristics described above.

### **3. STORY 3: BOHR *et al.* VERSUS EINSTEIN *et al.***

In this subtitle, the “*et all*” in the case of Niels Bohr means Arnold Sommerfeld (I think), the “Goettingen Trio” (Max Born, Werner Heisenberg and Pascual Jordan), Wolfgang Pauli and others, while in the case of Albert Einstein it includes Max Planck, Louis de Broglie, Erwin Schroedinger, B. Podolsky, N. Rosen, David Bohm, Karl R. Popper and others. The word “versus” refers to the difference in their views of science. Bohr held to the view that in the microworld there is no certainty, i.e., that science is not deterministic, but probabilistic. On the contrary, Einstein believed that science is deterministic. Given an already proven universal principle plus all pertinent initial and boundary conditions, what will become of a known, present state of a particle or a system in the future can be predicted deterministically.

Einstein expressed his conviction in his reply to Born’s letter of November 30, 1926: “The theory<sup>4</sup> says a lot but does not really bring us any closer to the secret of the “old one”. I, at any rate, am convinced that He<sup>5</sup> is not playing dice.” Heresay has it, that to this Bohr replied that that may be so<sup>6</sup>, but God often throws the dice in a direction which we do not know.

In their paper of 1935 on a thought experiment which became more known as EPR<sup>7</sup> paradox, A. Einstein, B. Podolsky, and N. Rosen concluded that quantum mechanics is an abstraction that only gives an incomplete and fragmentary description of the underlying reality. To this N. Bohr answered that the measurement process is an unanalyzable whole which led to the conclusion that there is no meaning to the attempt to give a detailed

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<sup>4</sup> He meant **Quantum Theory**

<sup>5</sup> It was God whom he was referring to.

<sup>6</sup> i.e., that God does not play dice.

<sup>7</sup> EPR: Einstein – Podolsky – Rosen. Their paper “Can Quantum Mechanical Description of Physical Reality Be Considered Complete?” was published in (1935) *Phys. Rev.*, 47, 777.

description of how correlations of position and momentum are carried along by the motions of the parts of a many-body system.

At a certain stage of the debate, the scientific community gave the verdict that the Einstein “camp” was the loser. Einstein conceded that the other party was more consistent in their argument. But he said that he only lost a battle, while the war was not over. In the early fifties David Bohm joined the war on Einstein’s side. He rediscovered de Broglie pilot wave which had been “crushed” by Wolfgang Pauli at the Solvay Conference in Brussels, in 1927, where Einstein, who was there, too, did not rise up in defense of his protégé.

Using the ideas of de Broglie and Bohm, John S. Bell, in his paper “On the Impossible Pilot Wave”, adds to the wave function  $\psi(n, x, t)$  a particle position  $\mathbf{X}(t)$ . According to Bell, probability enters once only, in connection with initial conditions. Thereafter the joint evolution of  $\psi$  and  $\mathbf{X}$  is perfectly deterministic. Yet Einstein was not enthusiastic about the idea, probably due to the nonlocality of the quantum potential.

The nonlocality of quantum potential is shown in the experiments of French physicist, Aspect (1982), where even on a macroscopic scale of meters we may not consider parts of a physical problem separately. Basing his conclusion on the observation of the Melbourne-based astronomer, Robert Hanbury Brown, Dutch Physicist, Stefan Persijn (2004) extends the scale to the distance between a point on Earth and another point on Sirius B. Quantum mechanically this is obvious as a single wave function describes the process. As Egbert Boeker says, “One may not even say that the whole is more than the sum of its parts as the parts may not be considered as separate entities.

Concluding this story, I would like to return to Louis de Broglie. He said that neutrinos are not massless. This was first confirmed in 1998 at Super-Kamiokande<sup>8</sup>. De Broglie’s *La Mécanique Ondulatoire* (Wave Mechanics) of the pilot wave has as its consequence a double solution, one of which representing superluminal particle. Perhaps this superluminal particle would have been considered confirmed if the OPERA<sup>9</sup>

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<sup>8</sup> Kamiokande stands for Kamioka Neutrino Detection Experiment. It is a neutrino observatory located under Mount Kamioka near the city of Hida, Gifu Prefecture, Japan. It is 1,000m underground in the Mazume mine. The detector is a huge cylindrical tank full of ultra-pure water surrounded by over 11,000 photomultiplier tubes.

<sup>9</sup> OPERA is an acronym that stands for “**O**scillation **P**roject with **E**mulsion-**t** **R**acking **A**pparatus”

experiment to determine the velocity of muon neutrino in 2011 yielded correct result, as it was found that the neutrino's velocity was faster than that of light in free space. The experiment is a collaboration between CERN in Geneva Switzerland, and the Laboratori Nazionali del Gran Sasso (LNGS) in Gran Sasso, Italy. Neutrinos produced at the CERN SPS are beamed to LNGS along an underground baseline 730 km in length in a period of about 3 milliseconds. The result was  $(v - c)/c = 2.48 \pm 0.28$  (stat.)  $\pm 0.3$  (sys.)  $\times 10^{-5}$ . (Adam, T., 2011) But after repeating the experiment several times, finally in May 2012 the Gran Sasso Lab stated that the original OPERA result had been mistaken.

The war between probabilism and determinism is still going on. At the helm of the probabilism group is Stephen Hawking (UK), while the leading physicist on the determinism camp is Roger Penrose (also in UK).

In the afternoon on July 4<sup>th</sup>, 2012 I learned from a BBC report sent to me by our daughter (who lives in Norwich, UK) that the long-sought Higgs boson had just been discovered by the ATLAS team of researches using the LHC (Large Hadron Collider) at CERN. If the result of the experiment was not mistaken and a Higgs boson was indeed discovered, it was a really very important discovery, as the boson is the key to understanding how massive particles in the universe have come into being, i.e., supposedly due to locally broken symmetry in a highly energetic energy field.

According to Suharyo Sumowidagdo, an Indonesian researcher at CERN who is a member of the team that discovered the particle, much work has still to be done to ascertain whether the particle was really a Higgs boson, although a boson it certainly was, decaying into two photons plus four fermions. But even if it turns out to have been something else, the discovery of such a big particle<sup>10</sup> is indeed very important.

When told that Louis de Broglie had thought of pilot wave associated with massless as well as massive particle, Einstein said, "It is a crazy idea ...." Similarly, when Peter Higgs in 1964 submitted his original paper to the European, journal, *Physics Letters*, the editors of the journal must have thought that the idea of a broken symmetry giving rise to mass was a crazy idea. Peter Higgs' paper was rejected by *Physics Letters*.

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<sup>10</sup> It is about 125.3 GeV of "weight", or 133 times heavier than proton.

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