PERCEPTUAL QUALITY OF GEOMETRICALLY DISTORTED IMAGES
PART II:
HUMAN PERCEPTION OF THE QUALITY OF GEOMETRICALLY DISTORTED IMAGES

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Abstract

This paper presents a study of how human perceive the quality of geometrically distorted images by presenting the design and analysis of a subjective-test experiment. The results of this experiment is then used to evaluate the performance HPQM (Homogeneity-based Perceptual Quality Measurement) method presented in the first paper of this series.

Keywords: Geometric distortion, human visual system, perceptual quality measurement of images, paired-comparison experiment

1. Introduction

Research on human perception of image quality has been widely performed. Aspects of images considered in such research are, for example, color, granularity or sharpness. Another example is to test specific artifacts of a compression algorithm (e.g., the blocking artifact of JPEG compression) or watermarking system (e.g., the random noise artifact of noise-based watermark-ing systems). Some examples of the image quality assessment for these distortions can be found in [1]. As a result, we already have a good understanding of how these aspects influence human perception of quality and we are able to quantify these perceptual aspects in cases where the distortion is near the visibility threshold. We can use the result, for example, to build a system to objectively measure image quality based on these aspects which corresponds quite well to subjective quality perception. We can also use the result of this research to improve the performance of various applications dealing with images by designing the systems such that most changes or distortions to the images occur in the areas that have small perceptual impact for human observers. The examples mentioned above, namely the compression algorithms and watermarking systems, are two examples of applications that can take advantage of this knowledge. However, the research on human perception of image quality has not dealt with another type of distortion that an image can undergo, namely geometric distortion (i.e., distortions due to geometric operations). As a result, we are currently unable to quantify the perceptual impact of geometric distortions on images.

This paper presents a study of the impact of geometric distortions on human perception of the quality of the affected images. The aim of this study is to provide both a better understanding of human perception of geometric distortion and a reference point with which to evaluate the performance of our novel objective geometric distortion measure scheme, the HPQM (Homogeneity-based Perceptual Quality Measurement) method, described in the previous paper of this series (see also [2]).

In order to perform this study we propose a user test system that is specifically designed to observe the impact of geometric distortion on human perception of image quality. The results we obtain from this test can also be useful to other researchers performing similar research in the fields of watermarking, image processing and human visual systems. Therefore, we have also made our test set and test results available for download on our website [3].

The rest of this paper is organized as follows. In Section 2, we present the design of our user test experiment and statistical analysis methods used to process the test results. In Section 3, we present the actual setup of our user test. In Section 4, we present and analyze the result obtained from this user test. In Section 5, we will briefly review our objective geometric distortion measure algorithm, present scores obtained using this method and evaluate its performance based on the subjective test result and compare its performance with other possible objective perceptual quality measurement systems. Finally, in Section 6, we present our conclusions and provide an outlook for further research.

2. Test design & analysis method

In this section we shall discuss in more detail the test design and the analysis tools we use to analyze the test results. The test design and analysis tools we use are well known in the literature [4, 6]. They have been used, for example, in experiments to determine consumer preference to certain products or product variants (e.g., different flavors of food) [4]. However, their usage in evaluating perceptual impact of geometric distortions in images, to the best of the author's knowledge, is novel and has never been discussed in the literature.

2.1. Test design

In order to evaluate the perceptual impact of geometric distortion, we performed a subjective test involving a panel of users, who are asked to evaluate a test set comprised of an original image and various distorted versions of it. The test subjects evaluate one pair of images at a time, comparing 2 images and choosing the one they think is more distorted. This type of experiment is called the paired comparison test. There are two experiment designs for a paired comparison test, namely the *balanced* and *incomplete* designs [4, 5]. In a balanced design, a test subject has to evaluate all possible comparison pairs taken from the test set. In the incomplete design, a test subject only has to perform comparisons of part of the complete test set. The latter design is useful when the number of objects in the test set is very large. In our experiment, we used the balanced paired-comparison design. Our choice for this design is based on three factors. Firstly, the number of objects in our test set is not very large and a test subject can finish the test within a reasonable time frame (as a rule of thumb, we consider a test lasting 60 minutes or less to be reasonable). Secondly, by asking every test subject to evaluate all objects in the test set we will be able to get a more complete picture of the perceptual quality of the images in the test set. Finally, in this design we make sure that each test subject evaluates an identical test set. This makes it easier to evaluate and compare the performance of each test subject.

Let t be the number of objects in the test set. One test subject performing all possible comparisons of 2 objects A_i and A_j from the test set, evaluating each pair once, will make $\binom{t}{2}$ paired comparisons in total. The result of the comparisons is usually presented in a $t \times t$ matrix. If ties are not allowed (i.e., a test subject must cast his/her vote for one object of the pair), the matrix is also called a ***two-way preference matrix*** with entries containing 1's if the object was chosen and 0's otherwise. An example of such a matrix for $t = 4$ is shown in Figure 1. Each entry $A_{i,j}$ of the matrix is interpreted as ***object A_i is preferred to object A_j***. The indices ***i*** and ***j*** refer to the rows and columns of the matrix, respectively.

	A_1	A_2	A_3	A_4
A_1	×	1	1	**0**
A_2	0	×	1	**1**
A_3	0	0	×	**0**
A_4	**1**	**0**	**1**	×

Figure 1. An example of a preference matrix

Let a_i be the number of votes object A_i received during the test. In other words, $a_i = \sum_{\substack{j=1,\\ i \neq j}}^{t} A_{i,j}$. We call a_i the *score* of object A_i. It is easy to see that the total score for all objects is

$$\sum_{i=1}^{t} a_i = \frac{1}{2} t(t-1) \tag{1}$$

and that the average score among all objects is

$$\bar{a} = \frac{\sum_{i=1}^{t} a_i}{t} = \frac{1}{2}(t-1) \tag{2}$$

We can extend these results to the case where we have ***n*** test subjects performing the paired comparison test. In this case, the test result can also be presented in a preference matrix similar to the one presented in Figure 1. However, each entry $A_{i,j}$ of this matrix now contains the number of test subjects who prefer object A_i to object A_j. If again we do not allow ties, the values of $A_{i,j}$ will be integers ranging from 0 to ***n***. We also note that in this case $A_{j,I} = n - A_{i,j}$. Finally, in this case, the total and average scores are expressed as $\frac{1}{2} nt(t-1)$ and $\frac{1}{2} n(t-1)$, respectively.

2.2. Statistical analysis of the experiment

After performing paired comparison tests, we obtain a preference matrix for each test set. Now we have to perform an analysis of this test result. We have two main objectives for this analysis. In the first place, we want to obtain the overall ranking of the test objects. The second objective is to see the relative quality differences between the test objects, that is, whether object A_i is perceived to be either similar to or very different in quality from object A_j. The analyses we perform on the data to achieve these objectives are the *coefficient of consistency*, the *coefficent of agreement* and the *significance test on score differences*. Each of these analyses is discussed in the following sections.

2.2.1. Coefficient of consistency

A test subject is consistent when he/she, in evaluating three objects A_x, A_y and A_z from the test set, does not make a choice such that A_x **à** A_y **à** A_z but A_z **à** A_x. The arrows can be interpreted as “preferred to”. Such a condition is called a *circular triad*. While circles involving more than three objects are also possible, any such circles can easily be broken up into two or more circular triads. The preference matrix presented in Figure 6.1 has one such triad, namely A_1 **à** A_2 **à** A_4 but A_4 **à** A_1.

For smaller values of t, one can easily enumerate the circular triads encountered. For larger t, this task becomes very tedious. However, we can compute the number of circular triads, c, from the scores a_i using the following relation [4,6]:

$$c = \frac{t}{24}(t^2 - 1) - \frac{T}{2} \tag{3}$$

where

$$T = \sum_{i=1}^{t} (a_i - \bar{a})^2 \tag{4}$$

The number of circular triads c can be used to define a measure of consistency of the test subjects. There are different approaches to do this [4]. Kendall/Babington-Smith compared the number of circular triads found in the test to the maximum possible number of circular triads. The coefficient of consistency ζ is defined as follows:

$$z = 1 - \frac{24c}{t(t^2 - 1)}, \text{ if } t \text{ odd} \tag{5a}$$

$$z = 1 - \frac{24c}{t(t^2 - 4)}, \text{ if } t \text{ even} \tag{5b}$$

There are no inconsistencies if, and only if, ζ = 1. This number will move to zero as the number of circular triads, thus the inconsistencies, increases.

The coefficient of consistency can be used in the following ways. In the first place, we can use this coefficient to judge the quality of the test subject. Secondly, we can use this coefficient as an indication of the similarity of the test objects. If, on average, the test ***subjects*** are inconsistent (either for the whole data set or a subset thereof), we can conclude that the test ***objects*** being evaluated are very similar and thus it is difficult to make a consistent judgement. Otherwise, if one particular test ***subject*** is inconsistent while the other test subjects are – on average – consistent, we may conclude that this particular subject is not performing well. If the consistency of this subject is significantly lower than average, we may consider removing the result obtained by this subject from further analysis.

2.2.2. Coefficient of agreement

The coefficient of agreement shows us the diversity of preferences among n test subjects. Complete agreement is reached when all n test subjects make identical choices during the test. From Section 2.1, we see that if every subject had made the same choice during the test (in other words, if there has been complete agreement), then half of the entries in the preference matrix will be equal to n, while the other half would be zero. Alternatively, in the worst case situation, all entries will be equal to $n/2$ (if n is even) or $(n \pm 1)/2$ if n is odd.

It is obvious that the minimum number of test subjects, n, that we need in order to be able to measure agreement is 2. Each time 2 test subjects make the same decision regarding a pair of test objects A_i and A_j, we say that we have one agreement regarding this pair. In other words, we measure the agreement by counting the number of pairs of test subjects that make the same decision about each pair of test objects. We do this by computing τ, defined as

$$t = \sum_{i=1}^{n} \sum_{j=1}^{n} \binom{A_{ij}}{2}, \qquad i \neq j \tag{6}$$

In Equation (6), $\binom{A_{ij}}{2}$ gives us the number of pairs of test subjects making the same choice regarding objects A_i and A_j. Thus τ gives us the total number of agreements among n test subjects evaluating t objects. Obviously, when $A_{i,j} = 1$ we do not have any agreement among the subjects and the contribution of this particular $A_{i,j}$ to τ would be zero. If $A_{i,j} = 0$, it means that all test subjects agree ***not*** to choose A_i over A_j. Although the contribution of this $A_{i,j}$ to τ is also zero, the number of agreements regarding this pair of test objects will be reflected by the value of $A_{j,i}$.

We have $\binom{t}{2}$ pairs of comparisons and $\binom{n}{2}$ possible pairs of subjects, therefore the maximum number of agreements between the subjects is given by

$$\max(t) = \binom{t}{2}\binom{n}{2} \tag{7}$$

Meanwhile, the minimum value of τ is given by

$$\min(t) = \binom{t}{2}\binom{\lfloor n/2 \rfloor}{2} \tag{8}$$

We can also express τ in a more computationally convenient way, as follows.

$$t = \frac{1}{2}\left[\sum_{i \neq j} a_{i,j}^{2} - n\binom{t}{2}\right] \tag{9}$$

Kendall/Babington-Smith [6] defines the coefficient of agreement, u, as follows

$$u = \frac{2t}{\max(t)} - 1 = \frac{2t}{\binom{t}{2}\binom{n}{2}} - 1 \tag{10}$$

The value of u = 1 if and only if there is complete agreement among the test subjects, and it decreases when there is less agreement among the test subjects. The minimum value of u is -1/(n-1) if n is even or -1/n if n is odd. The lowest possible value of u is -1 which can only be achieved when n = 2. This value of u shows the strongest form of disagreement between the test subjects, namely that the test subjects completely contradict each other.

We can perform a hypothesis test to test the significance of the value u. The null hypothesis is that all test subjects cast their preference completely at random. The alternative hypothesis is that the value of u is greater than what one would expect if the choices would have been made completely at random. To test the significance of u we use the following statistic, as proposed in [4]

$$X^2 = \frac{4}{n-2}\left[t - \frac{1}{2}\binom{t}{2}\binom{n}{2}\frac{(n-3)}{(n-2)}\right] \tag{11}$$

which has χ^2 distribution with $\binom{t}{2}\frac{n(n-1)}{(n-2)^2}$ degrees of freedom.

As n increases, the expression in Equation (11) reduces to a simpler form [7]

$$X^2 = \binom{t}{2}[1 + u(n-1)] \tag{12}$$

with $\binom{t}{2}$ degrees of freedom.

It is important to note that consistency and agreement are two different concepts. Therefore, a high u value does not necessarily imply the absence of inconsistencies and vice versa.

The coefficient of agreement also shows whether the test objects, on average, received equal preference from the test subjects. If the overall coefficient of agreement is very low we can expect that the score of each test object will be very close to the average scores of all test objects, i.e., there is no significant difference among the scores. As a consequence, assigning ranks to the objects or drawing the conclusion that one object is better (or worse) than the others is pointless since the observed score differences (if any) cannot be used to support the conclusion. On the other hand, strong agreement among the test subjects indicates that there exist significant differences among the scores.

2.2.3. Significance test of the score difference

A significance test of the score difference is performed in order to see whether the perceptual quality of any 2 objects from the test set is perceived as different. In other words, the perceptual quality of object A_i is declared to be different from the quality of object A_j, only if a_i is significantly different from a_j. Otherwise, we have to conclude that the test subjects consider the perceptual quality of the 2 objects to be similar.

This problem is equivalent to the problem of dividing the set of scores we obtain, i.e. $S = \{a_1, a_2, \ldots, a_t\}$, into sub-groups such that the variance-normalized *range* (the difference of the largest and lowest values) of the scores within each group,

$$R = \frac{(a_{\max} - a_{\min})}{s_{a_i}} \tag{13}$$

is lower or equal to a certain value $\lceil R_c \rceil$ (in other words, the difference of any 2 scores within the group must be lower or equal to $\lceil R_c \rceil$), which depends on the value of the significance level α. In other words, we want to find R_c such that the probability $P[R \geq R_c]$ is lower or equal to the chosen significance level α. We declare the objects within each group to be not significantly different, while those from different groups are declared to be significantly different. By adjusting the value of α, we can adjust the size of the groups. This in turn controls the probability of false positives (declaring 2 objects to be significantly different when they are not) and false negatives. The larger the groups, the higher the probability of false negatives. On the other hand, the smaller the groups, the higher the probability of false positives.

The distribution of the range R is asymptotically the same as the distribution of variance-normalized range, W_t, of a set of normal random variables with variance = 1 and t samples [4]. Therefore, we can use the following relation to approximate $P[R \geq R_c]$

$$P[W_{t,a} \geq \frac{2R_c - \frac{1}{2}}{\sqrt{nt}}] \tag{14}$$

In Equation (14), $W_{t,\alpha}$ is the value of the upper percentage point of W_t at significance point α. The values of $W_{t,\alpha}$ are tabulated in statistics books for example the one provided in [8].

The significance test for the differences between scores proceeds as follows:

1. Choose the desired significance level α.
2. Compute the critical value R_c using the following relation
$$R_c = \left\lceil \frac{1}{2} W_{t,a} \sqrt{nt} + \frac{1}{4} \right\rceil \tag{15}$$
3. Any difference between 2 scores that is lower than R_c is declared to be insignificant. Otherwise, the score difference is declared significant.

3. Test procedure

User test mechanism to measure the impact of geometric distortion on the human perception of image quality is not widely discussed in the literature. Therefore, we have proposed a new user test system that is specifically designed for this purpose. In this section we shall describe in more detail the design of a suitable test set and user interface for such user test system.

3.1. Test set

We used the same test sets as the ones we used in the previous paper in this series. The two images used as a basis to build our test set, i.e. the Bird and Kremlin images (shown in Figure 2), are 8-bit grayscale images with 512 × 512 pixels resolution. The images are chosen primarily due to their content. The first image, Bird, does not have many structures such as straight lines. Furthermore, not every test subject is very familiar with the shape of a bird (in particular the species of bird depicted in the image). So in this case, a subject should have little (if any) "mental picture" of what things should look like. On the other hand, the Kremlin picture has a lot of structures and even though a test subject may not be familiar with the Kremlin, he/she should have some prior knowledge of what buildings should look like.

(a) *(b)*

Figure 2. The 2 basis images: (a) Bird and (b) Kremlin

We used 17 different versions of the images. Each version is geometrically distorted in a different way. Thus in our test we use t = 17. The geometric distortions we used in the experiment are also identical to the ones used in the previous paper and their descriptions are repeated here in Table 1 for convenience. As in the previous paper, we use the notation A_i, with i = 1, 2, … 17 to identify each image.

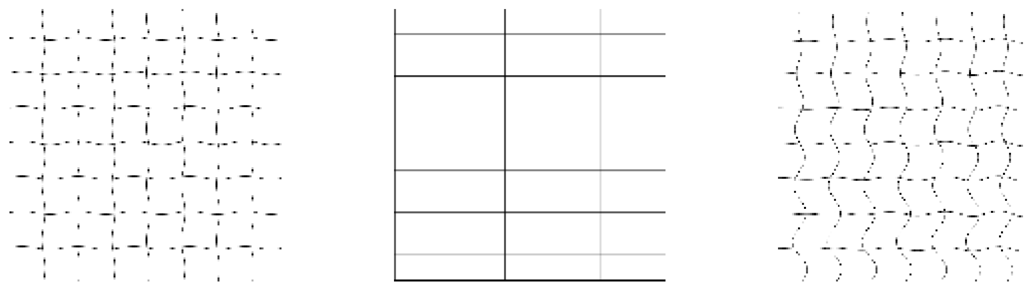
The distortions chosen for the test set range from distortions that are perceptually not disturbing to distortions that are easily visible. The global bending distortions $\{A_6, A_7, A_8, A_9\}$ are chosen because these kinds of distortions are, up to a certain extent, visually not very disturbing in natural images. However, this distortion severely affects the PSNR value of the distorted images. The sinusoid (stretch-shrink) distortions $\{A_{10}, A_{11}, A_{12}, A_{13}\}$ distort the image by locally stretching and shrinking the image. Depending on the image content, this kind of distortion may not be perceptually disturbing. The rest of the distortions distorts the image by shifting the pixels to the left/right or upwards/downwards. These distortions are easily visible, even when the severity is low. The distortions $\{A_2, A_3, A_4, A_5\}$ apply the same distortion severity over the whole image, while the severity of distortions $\{A_{14}, A_{15},$

A_{16}, A_{17}} is varied within the image. Some examples of the geometric distortions used in the experiment are shown in Figure 3.

Table 1. Geometric distortions used in the experiment

Image	Description
A_1	No distortion (original image)
A_2	Sinusoid, amplitude factor = 0.2, 5 periods
A_3	Sinusoid, amplitude factor = 0.2, 10 periods
A_4	Sinusoid, amplitude factor = 0.5, 5 periods
A_5	Sinusoid, amplitude factor = 0.5, 10 periods
A_6	Global bending, bending factor = 0.8
A_7	Global bending, bending factor = - 0.8
A_8	Global bending, bending factor = 3
A_9	Global bending, bending factor = -3
A_{10}	Sinusoid (stretch-shrink), scaling factor 1, 0.5 period
A_{11}	Sinusoid (stretch-shrink), scaling factor 1, 1 period
A_{12}	Sinusoid (stretch-shrink), scaling factor 3, 0.5 period
A_{13}	Sinusoid (stretch-shrink), scaling factor 3, 1 period
A_{14}	Sinusoid (increasing freq), amplitude factor = 0.2, starting period = 1, freq increase factor = 4
A_{15}	Sinusoid (increasing freq), amplitude factor = 0.2, starting period = 1, freq increase factor = 9
A_{16}	Sinusoid (increasing amplitude), start amplitude factor = 0.1, 5 periods, amplitude increase factor = 4
A_{17}	Sinusoid (increasing amplitude), start amplitude factor = 0.1, 5 periods, amplitude increase factor = 9

We then proceed to make all possible comparison pairs out of the 17 images, including the comparison of an image with itself. In each pair, we designate the first image as the left image and the other as the right image. This refers to how the images are to be presented to the subjects (see Figure 4). We then repeat each pair once, with the left-right ordering of the images reversed. Thus we have 306 pairs of images for each of the two images for a total of 612 pairs of images in the test set.



(a) *(b)* *(c)*

Figure 3. Examples of the geometric distortions:
(a) Distortion A_5, (b) Distortion A_{13} and (c) Distortion A_{16}

3.2. Test subjects

The user test experiment involved 16 subjects, consisting of 12 male (IL, ON, PD, AH, ES, DS, IS, JO, JK, JJ, KK and RH) and 4 female (KC, CL, CE and ID) subjects. The subjects have different backgrounds and levels of familiarity with the field of digital image processing. As discussed in Section 3.1, each user will examine each pair of test images twice in one test session. Furthermore, subjects IL, DS and IS each perform 3 test sessions. Therefore, in the tables found in Section 4, a number will be added to the subject names to show different test sessions (eg., IL1 shows the result of subject IL from the 1st test, etc.). These repetitions are done to see the difference between test results for one person when the test is repeated. We assume that each repetition of the test (both within a single test session and between test sessions) is independent. Therefore, we have the total number of test repetitions n = 44.

3.3. Test procedure

The test is performed on a PC with a 19-inch flatscreen CRT monitor. The resolution is set at 1152 × 864 pixels. The vertical refresh rate of the monitor is set at 75 Hz. To perform the test, a graphical user interface is used. This user interface is shown in Figure 4.

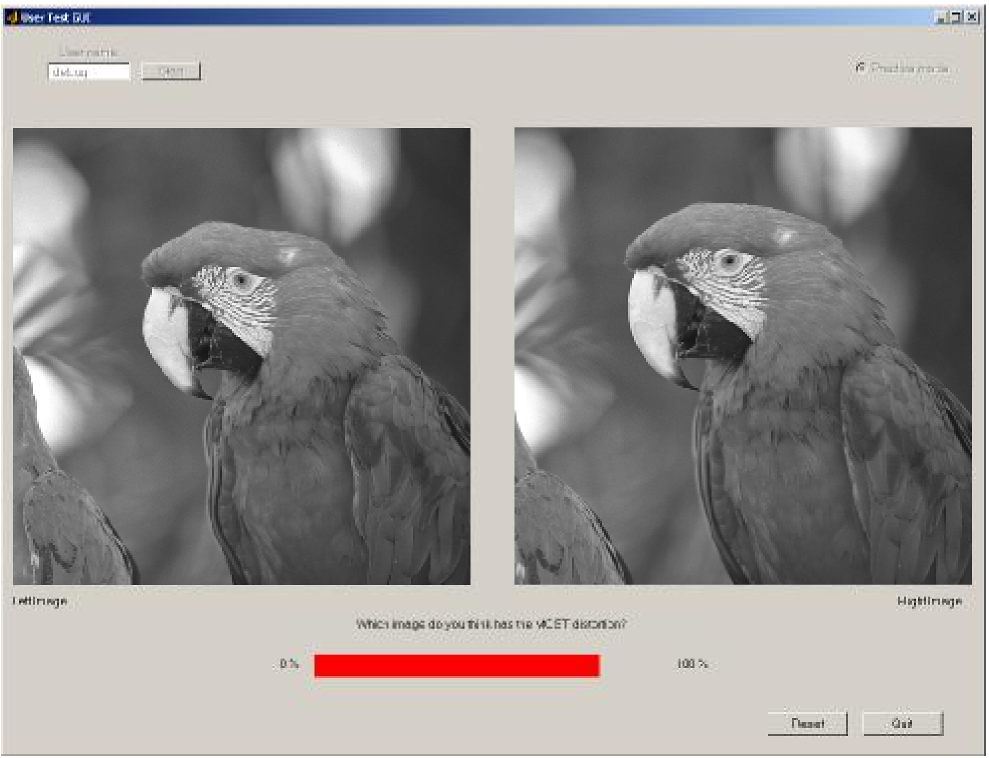


Figure 4. The user interface used in the experiment

4. Test results and analysis

4.1. User preference matrix

After performing the user test, we obtain the preference matrices for the Bird and Kremlin images. In Tables 2(a) and 2(b), we show the preference matrices obtained for the Bird and Kremlin test images. These preference matrices are available for downloading from our website [3]. The images codes refer to Table 1. The column a_i shows the sum of each row, i.e., the score of each image A_i. Since in our experiment the test subject is asked to choose the image with the *most* distortion, a smaller score a_i means that the image is perceptually better.

Table 2(a). Preference matrix for the Bird image

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	a_i
A_1	×	7	3	0	0	11	21	19	8	8	24	2	10	9	1	0	0	123
A_2	37	×	4	0	0	33	28	29	24	30	36	10	11	12	5	1	1	261
A_3	41	40	×	3	1	42	43	39	39	41	42	25	18	37	9	5	0	425
A_4	44	44	41	×	3	44	44	42	43	43	43	37	39	43	31	15	1	557
A_5	44	44	43	41	×	44	44	44	43	43	44	42	43	44	43	42	24	672
A_6	33	11	2	0	0	×	25	15	17	15	33	4	11	8	1	0	0	175
A_7	23	16	1	0	0	19	×	15	13	21	28	3	11	5	2	0	0	157
A_8	25	15	5	2	0	29	29	×	12	17	27	6	10	9	1	1	0	188
A_9	36	20	5	1	1	27	31	32	×	30	40	8	15	15	2	2	0	265
A_{10}	36	14	3	1	1	29	23	27	14	×	34	6	9	9	0	0	0	206
A_{11}	20	8	2	1	0	11	16	17	4	10	×	4	5	6	1	0	0	105
A_{12}	42	34	19	7	2	40	41	38	36	38	40	×	20	31	9	5	1	403
A_{13}	34	33	26	5	1	33	33	34	29	35	39	24	×	25	17	5	0	373
A_{14}	35	32	7	1	0	36	39	35	29	35	38	13	19	×	6	1	0	326
A_{15}	43	39	35	13	1	43	42	43	42	44	43	35	27	38	×	7	2	497
A_{16}	44	43	39	29	2	44	44	43	42	44	44	39	39	43	37	×	1	577
A_{17}	44	43	44	43	20	44	44	44	44	44	44	43	44	44	42	43	×	674

4.2. Statistical analysis of the preference matrix

4.2.1. Coefficient of consistency (ζ)

We measured the coefficient of consistency for individual test subjects using Equation (5a) since we have t = 17. Since each test subject performs the user test twice per session, we use the average value of ζ as an indication of each subject's consistency. The average coefficient of consistency is presented in Table 3.

Table 3. Coefficient of consistency (ζ)

Subject	Bird	Kremlin	Subject	Bird	Kremlin
IL1	0.83	0.93	DS1	0.67	0.87
IL2	0.83	0.91	DS2	0.73	0.92
IL3	0.85	0.95	DS3	0.82	0.93
KC	0.85	0.86	IS1	0.92	0.95
ON	0.94	0.98	IS2	0.94	0.93
PD	0.70	0.87	IS3	0.94	0.97
AH	0.87	0.96	JO	0.93	0.97
CL	0.82	0.90	JK	0.90	0.96
CE	0.83	0.94	JJ	0.85	0.88
ES	0.89	0.94	KK	0.70	0.79
ID	0.66	0.90	RH	0.90	0.95

From Table 3 we can conclude that in general the test subjects are consistent in their decisions. We can also see that in general the values of ζ for the Bird image are lower than those of the Kremlin image. This is due to the fact that the Kremlin image contains more structure compared to the Bird image, which helps the test subjects to make consistent decisions. Furthermore, the unfamiliarity of the test subjects with the particular species of bird depicted in the image also makes it difficult to make consistent decisions.

4.2.2. Coefficient of agreement (u)

We measured two types of coefficient of agreement from the preference matrix. The first is the overall coefficient of agreement that measures the agreement among all test subjects in the experiment. The second is the individual coefficient of agreement that measures the agreement of a test subject with him-/herself during the two repetitions in a test session. A low u value in this case would indicate that the subject is confused and does not have a clear preference for the images being shown.

For the overall coefficient of agreement, we have n = 44 and t = 17. For these values, the maximum and minimum values of u are 1 and -0.0227, respectively. From the preference matrices, we can calculate that the overall coefficient of agreements are

u_{bird} = 0.574 and $u_{kremlin}$ = 0.731. Performing the significance test on both u values using the method described in Section 2.2.2 shows that in both cases the value of u is significant at α = 0.05. Therefore, we can conclude that in both cases there are strong agreements among the test subjects. However, we can also see that the agreement in the case of the Bird image is much weaker than the Kremlin image, due to the image content.

Table 4. Individual Coefficient of Agreements(u)

Subject	Bird	Kremlin	Subject	Bird	Kremlin
IL1	0.559	0.750	DS1	0.265	0.647
IL2	0.574	0.721	DS2	0.471	0.794
IL3	0.677	0.779	DS3	0.559	0.750
KC	0.662	0.691	IS1	0.721	0.882
ON	0.779	0.868	IS2	0.809	0.721
PD	0.485	0.677	IS3	0.735	0.838
AH	0.559	0.794	JO	0.721	0.853
CL	0.456	0.750	JK	0.824	0.735
CE	0.618	0.691	JJ	0.529	0.691
ES	0.765	0.765	KK	0.368	0.515
ID	0.279	0.691	RH	0.691	0.765

For the individual coefficient of agreement, we have n = 2 and t = 17. In this case, we have $-1 \leq u \leq 1$. The individual coefficient of agreements are presented in Table 4. As expected, we see that all subjects have larger u values for the Kremlin image. The exceptions to this are subject ES, who has the same u values for both images, and subjects IS2 and JK, who have larger u for the Bird image. After performing the significance test on the values of u, we can conclude that all subjects have u values that are significant at α = 0.05 for both the Bird and Kremlin images.

4.2.3. Significance test of score differences

The strong agreements among the test subjects for both images, as shown in the previous section, show that there exist significant differences among the scores of the test objects. We use the procedure described in Section 2.2.4 to find the critical value for the score difference for the images, at significance level α = 0.05. From [8] we have $W_{t,\alpha}$= 4.89. Substituting this value into Equation (15), we have R_c = 67.12 and thus we set R = 68. Therefore, only objects having a score difference of more than 68 are to be declared significantly different.

In Figure 5, we present the grouping of the images in the test set based on the significance of the score differences. The images have been sorted from left to right based on their scores, starting from the image with the smallest score (i.e., perceived to have the highest quality) to the one with the largest score. The score for each image is shown directly under the image code. Images having a score difference smaller than 68 are grouped together. This is represented by the shaded boxes under the image code. For example, in Figure 5(a), images A_{14} and A_{13} belong to one group.

A_1	A_1	A_7	A_6	A_8	A_1	A_2	A_9	A_1	A_1	A_1	A_3	A_1	A_4	A_1	A_5	A_1
1					0			4	3	2		5		6		7
10	12	15	17	18	20	26	26	32	37	40	42	49	55	57	67	67
5	3	7	5	8	6	1	5	6	3	3	5	7	7	7	2	4

(a)

A_1	A_1	A_1	A_6	A_1	A_7	A_1	A_8	A_2	A_1	A_9	A_3	A_1	A_4	A_1	A_1	A_5
	0	2		1		3			4			5		6	7	
85	94	10	11	15	20	27	31	34	38	39	48	49	57	59	67	68
		8	3	4	2	4	5	8	1	9	9	5	9	2	5	1

(b)

Figure 5. Score grouping for: (a) Bird image and (b) Kremlin image

From Figure 5, we can see that the images occupying the last 6 positions of the ranking for both the Bird and Kremlin images are distorted using the same distortion. Furthermore, they are sorted in the same order (except for images A_5 and A_{17}, but the difference between their scores is not significant). Thus we can conclude that these distortions are perceived similarly by the test subjects, regardless of the image content. These distortions occupy the "lower quality" segment of the ranking so we can also conclude that the distortions are so severe that the image content no longer plays a significant role. For the other images, the influence of image content on the perceived quality of the distorted images is larger.

Table 5. Group u values

Bird			Kremlin		
Group	u	Significant?	Group	u	Significant?
$A_{11}A_1A_7$	0.006	No	$A_1\ A_{10}\ A_{12}\ A_6$	0.008	No
$A_1A_7A_6A_8$	0.061	Yes	$A_{10}\ A_{12}\ A_6\ A_{11}$	0.03	Yes
$A_7A_6A_8A_{10}$	0.041	Yes	$A_{11}\ A_7$	0.011	No
$A_{10}A_2A_9$	0.07	Yes	$A_{13}\ A_8$	0.08	Yes
$A_2A_9A_{14}$	0.085	Yes	$A_8\ A_2\ A_{14}$	0.175	Yes
$A_{14}A_{13}$	-0.004	No	$A_2\ A_{14}\ A_9$	0.054	Yes
$A_{13}A_{12}A_3$	-0.003	No	$A_3\ A_{15}$	-0.021	No
$A_{15}A_4$	0.148	Yes	A_4A_{16}	0.112	Yes
A_4A_{16}	0.08	Yes	$A_{17}\ A_5$	0.011	No
A_5A_{17}	-0.015	No	-	-	-

Table 5 shows the overall u values for each score group. We expect that when the images in a group do not have significantly different scores, there will not be any clear preference for any of them among the test subjects and therefore the u values should be low. The groups are presented in the 1st and 4th columns using their members as group names. The 3rd and 6th columns of the table show the result of the significance test for u, as described in Section 2.2.2, with significance level $\alpha = 0.05$.

We can conclude from Table 5 that the u values for each group are very low. Some groups even have u values that are not significantly larger than the u values that would have been achieved had the votes within that group been cast at random. This result shows that indeed the grouping of the images performed based on the significance of score differences has produced groups within which the perceived quality is difficult to distinguish.

4.3. Conclusions

From the analysis of the user test results, we can draw the following conclusions:

1. The test objects are generally perceptually distinguishable by the test subjects. This is supported by the fact that the consistency of the test subjects is relatively high, as shown in Table 3. Furthermore, we also see that the individual ***u*** values (shown in Table 4) are also high.
2. There is a general agreement as to the relative perceptual quality of the test images among the test subjects. This is supported by both the high overall and individual ***u*** values. Therefore, we can make a ranking of the images based on their perceived quality.
3. For some images, the relative perceptual quality among them is not clearly distinguishable. We can see this from the grouping of the scores based on the significance test of score differences. This is further supported by the lack of agreement among test subjects regarding the relative quality of images within such groups.

5. Evaluation of the objective perceptual quality measurement method

5.1. Overview of the method

The objective geometric distortion measurement is based on the ideas in our previously published work [9] and further developed and described in [2] and in the previous paper of this series. The algorithm is based on the hypothesis that the perceptual quality of a geometrically distorted image depends on the homogeneity of the geometric distortion. We call our proposed scheme the Homogeneity-based Perceptual Quality Measurement (HPQM). The less homogenous the geometric distortion is, the lower the perceptual quality of the image will be. We proposed a method to measure this homogeneity by approximating the underlying geometric distortion using simple RST approximation. We use the Optical Flow Estimation (OFE) algorithm [10,11] to perform this approximation. We increase the locality of our approximation until the level of approximation error is lower than a predetermined threshold or until the locality of the approximation reaches a predetermined maximum. The locality is increased using quadtree partitioning of the image, where smaller block sizes indicate higher approximation locality. We then determine the score (i.e., the quality) of the image based on the resulting quadtree structure. In the objective test, the score that can be achieved by an image is normalized to the range of 0 – 100.

5.2. Performance evaluation

In this section we shall evaluate the performance of our proposed objective quality measurement algorithm. In this performance evaluation we use the results of the subjective-test as a ground truth. In other words, the proposed algorithm will be considered to be performing well if its results have a good correspondence to the subjective-test results. Furthermore, in order to evaluate the performance of the proposed objective quality measurement algorithm relative to the performance of other possible measurement schemes, we also evaluate the performances of two other possible objective quality measurement schemes. The other possible measurement schemes we evaluate in this section are PSNR measurement and Motion-Estimation (ME)-based measurement scheme.

The PSNR measurement is a widely used tool used to evaluate the objective quality of images. Although this measurement does not always correspond well to human perception of quality, its performance is good enough to evaluate the quality of, for example, images degraded by additive noise. However, PSNR measurement relies heavily on the pixel-per-pixel correspondence between the images being evaluated. Since geometric distortion destroys this correspondence, PSNR measurement is not well suited for evaluating geometrically distorted images. Therefore, in our experiments the results of the PSNR measurement are used to indicate the worst-case scenario (i.e., an ineffective measurement scheme).

The second alternative objective quality measurement scheme we evaluate is an motion estimation (ME)-based measurement scheme. This measurement scheme is inspired by the use of motion estimation techniques in image and video watermarking to deal with geometric distortion for example the technique presented in [12]. In order to use the motion estimation technique as a measurement scheme we take into account two outputs of the motion estimation process, namely the motion vector entropy and the variance of the prediction error. The motion vector entropy is used to indicate the "activity" of the distortion. A high activity means that various parts of the image are distorted in a different way. The higher the activity of the distortion, the lower the perceptual quality of the image. The variance of the prediction error shows the residual error after the motion estimation and compensation process. A large error variance indicates a heavy distortion and thus a lower perceptual quality. Our observations indicate that the motion vector entropy plays a more important role in determining the perceptual quality of the image. Therefore, we give this measurement parameter a larger weight than the residual error variance. These weights are determined experimentally. The proposed ME-based quality measurement (MEQM) scheme is presented in Figure 6. In our experiments, we chose a block size of 16 × 16 pixels, maximum displacement of 7 pixels and full-search method. This ME-based measurement approach is somewhat similar to the HPQM approach with two main differences. The first difference between the two is the simpler approximation model of the MEQM scheme. The MEQM scheme uses only translation instead of an RTS/affine model used by HPQM. The second difference is in the locality of the approximation. The MEQM scheme uses a fixed locality for the approximation. This locality is determined by the chosen block size. In other words, we can regard the MEQM scheme as a simpler, more restricted, version of the HPQM.

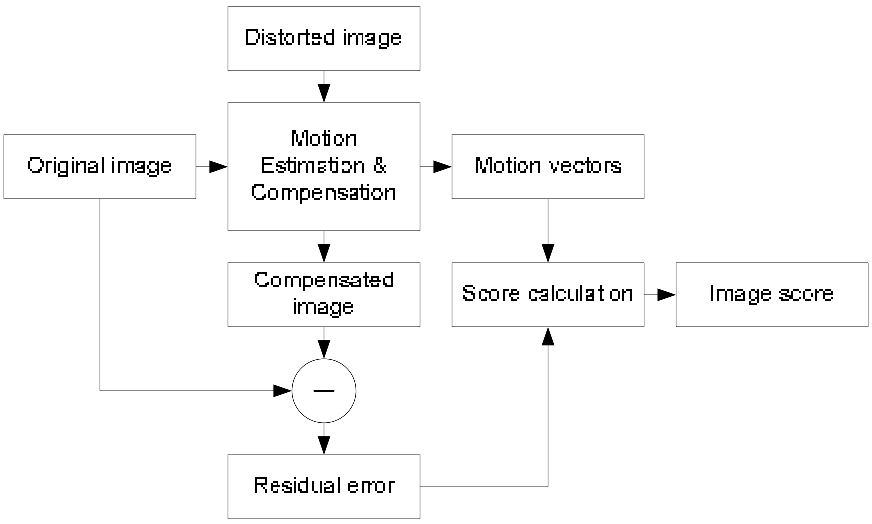


Figure 6. An ME-based measurement scheme

In evaluating the performance of the objective quality measurements, we look at the ***intra-*** and ***inter-distortion*** comparisons. For intra-distortion comparisons, we evaluate the scores of the images within one type of geometric distortion, but with different distortion parameters. For example, we perform an intra-distortion comparison by evaluating the scores of images A_2, A_3, A_4 and A_5 that are distorted by the same sinusoid distortion but with different parameters (see Table 1). In this comparison, an image with a more severe distortion parameters should get a lower score. For inter-distortion comparisons, we evaluate the scores of all images in the test set. This is a more difficult test for the objective quality measurement schemes since they have to be able to indicate the relative perceptual qualities between different types of geometric distortions.

All measurement schemes that we evaluated in our experiments, including PSNR measurement, perform well in the intra-distortion comparison. In other words, the images distorted with a more severe parameter set are correctly given lower scores. In order to evaluate the performance of the objective quality measurement schemes in performing inter-distortion comparisons, we plot their results against the subjective test scores. The comparison plots for the Bird image set is shown in Figure 7. The plots for the Kremlin image set show similar behavior.

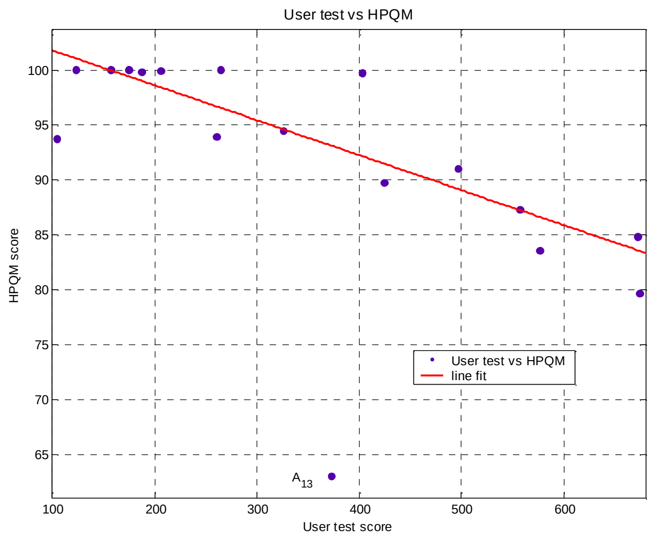


Figure 7. Result comparisons for the Bird image:
(a) User test vs. HPQM

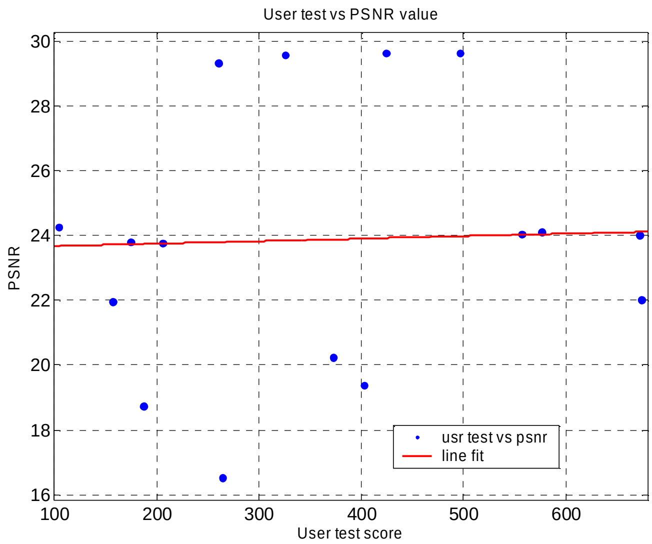
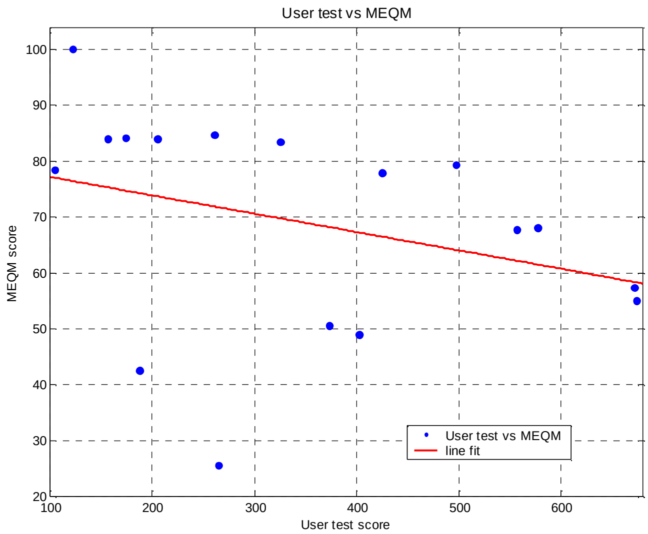


Figure 7. (continued):

(b) User test vs. PSNR



(c)

Figure 7. (continued):

(c) User test vs. MEQM

From Figure 7(b) we can see that the PSNR measurement has a very poor correspondence to the subjective test result. This is shown by the regression line that is virtually horizontal. The value of the correlation coefficient ρ in this case also reflects this fact, namely we have ρ_{up} = 0.14. The MEQM scheme performs much better than PSNR measurement as shown in Figure 7(c) and with ρ_{um} = -0.32. We can also see that the HPQM scheme gives the best performance among the three evaluated schemes, as shown in Figure 7(a) and with ρ_{uh} = -0.6. The negative values of ρ_{um} and ρ_{uh} correctly reflect the fact that in our experiments a larger subjective test score represents a lower perceptual quality.

If we evaluate Figure 7(a) we can see that image A_{13} does not properly fit the behavior of the rest of the data set and can be considered an outlier. Removing this image from the data set and recalculating the correlation coefficient, we get ρ_{uo} = -0.87. In general, we observe that the HPQM scheme cannot handle images distorted by the ***sinusoid (stretch-shrink)*** distortion (see Table 1) well, except for image A_{10}[1]. At present, we do not yet have a satisfactory explanation regarding this phenomenon. In the case of image A_{10}, the geometric transformation applied to this image is similar to the one implemented in television broadcasting when it is necessary to convert video frames from one aspect ratio to another. This transformation is perceptually not disturbing (unless there is a lot of movement, for example camera panning), and therefore, our test subjects give this image a high ranking. In this distortion, the image is stretched slightly in the horizontal and vertical direction. The slight increase in image width and height is compensated by shrinking the outer parts of the image. This distortion can be approximated by slightly enlarging the original image. Since this is a homogenous RTS approximation, the HPQM scheme gives this image a high score. Image A_{11} of the Bird test set is interesting since the subjects prefer this image to the undistorted image A_1. This is probably due to the unfamiliarity of the subjects to the bird species shown in the picture. Apparently, the test subjects get the impression that the size of the bird's head in the original image was either too large or too flat. Therefore, they preferred the image in which the head of the bird is slightly shrunk horizontally (and consequently slightly rounder). The fact that this does not happen in the Kremlin test set (see Table 2(b)) seems to support this conclusion.

6. Conclusion and future works

In this paper, we have described the method we use to perform a perceptual user test for geometrically distorted images. We also described the statistical tools we use to analyze the results of the user test. The result of the user test is then used as a

[1] Similarly, the MEQM scheme also seems to have difficulties in dealing with this type of distortions.

ground truth to validate our objective perceptual quality measurement scheme, the HPQM, which is based on the hypothesis that the perceptual quality of a distorted image depends on the homogeneity of the geometric transformation causing the distortion. Furthermore, in order to have a better assessment of the performance of the HPQM, we also compare its performance to the performance of the PSNR measurement and the MEQM scheme. In our experiments, we evaluate the performance of all three objective measurement schemes in two areas, namely in performing intra- and inter- distortion comparisons.

All objective measurements evaluated in our experiments, the HPQM, PSNR and MEQM, give similar performance in performing intra-distortion comparisons. For inter-distortion comparisons, the PSNR measurement performs poorly. The MEQM and HPQM schemes outperform PSNR measurement in this category, with the HPQM giving the best performance among the three evaluated schemes.

While the amount of data collected in our experiments is not yet large enough to form firm conclusions, we observe a very strong tendency that our HPQM scheme has a very good overall correspondence to the results of the subjective test. The scheme is not yet perfect, however, and we still observe some discrepancies between the ranking of the images generated by HPQM to that generated by the subjective test result.

In the future, more measurements and user test experiments similar to the one described and analyzed in this chapter should be performed. The data collected from such experiments can than be used to further validate or refine the hypothesis and to further fine-tune the performance of the HPQM scheme. Finally, other objective quality measurement approaches should also be explored and tested.

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